

Converting Neutron Stars into Strange Stars: Instanton Model

Victor Ts. Gurovich[†] and Leonid G. Fel[‡]

[†]Department of Physics, Technion, Haifa 32000, Israel

[‡]Department of Civil Engineering, Technion, Haifa 32000, Israel

Abstract

We estimate the quasiclassical probability of the homogeneous nuclear matter transition to a strange matter when a detonation wave propagates radially inside a sphere of nuclear matter. For this purpose we make use of instanton method which is known in the quantum field theory.

Keywords: Strange matter, Instanton.

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1 Introduction

It was first pointed out [3, 27] that strange matter (SM) composed of three quarks might be a ground state of a normal nuclear matter (NM) at zero temperature and pressure, which was later supported by studies based on MIT bag model [6]. A conversion of NM to SM is suppressed at ordinary nuclear densities. The existence of stable SM would have some remarkable consequences in cosmology and astrophysics. At very large densities of NM like those in neutron stars (NS), where the Fermi energy is higher than mass of s quark, the NM–SM transition may occur spontaneously. This led to conjecture [2, 3, 17, 27] that strange stars, which are predominantly made of SM, may be formed from dense NS. The conversion is assumed to be triggered at the core of NS [1, 21] where the density reaches values $2 \cdot 10^{14} \text{ g/cm}^3 < \rho_* < 6 \cdot 10^{15} \text{ g/cm}^3$ with a total mass of the star $M \geq 1.5 \odot$. There may appear stable SM drops, called *strangelets* [6], if every single drop possesses a baryon charge A exceeding some critical value A_* . Further growth of strangelets occur by outward diffusion of strange quarks to ambient NM [1, 21].

Equation of SM state has been suggested in [27], $P_s = (E_s - E_o)/3$, where P_s and E_s stand for the pressure and density of energy, and E_o denotes a density of energy of SM at zero pressure. If $E_s \gg E_o$

then transition from the non relativistic NM ($P_n \ll E_n$) to SM occurs with essential growth of pressure and temperature.

There are two different models which treat the NM–SM transition in framework of relativistic hydrodynamics: combustion waves (CW) [1, 21] and detonation waves (DW) [13, 25]. The CW propagates as a slow combustion with a speed $V_c \simeq 10^7 m/s$, while the DW propagates with $V_d \simeq 10^8 m/s$. In [25] DW was considered as the self-similar spherical wave propagating with a constant rate w.r.t. NM of constant density. Different aspects of this conversion were discussed in [13, 16].

The problem arises when the classical solution is considered at the strangelet scale with radius $R_s = (3Am_n/4\pi\rho)^{1/3}$, where ρ denotes a density of NM and m_n stands for neutron mass. For strangelets with baryon charge $A \simeq 10 - 100$ this radius varies in the range $R_s \simeq 1.2 - 2.5 \cdot 10^{-15} m$. On the other hand, the de Broglie wavelength $\lambda_B = h/(Am_n V_d)$ for the strangelet reads, $\lambda_B \simeq 0.4 - 4 \cdot 10^{-15} m$, *i.e.*, both λ_B and R_s have comparable values. This manifests the quasiclassical nature of the strangelets which trigger the NM–SM transition and poses a question about probability of such transition.

To answer this question we make use of the known in quantum theory instanton approach [23] which describes a tunneling between different field configurations. We calculate a probability of the NM–SM conversion when DW propagates spherically inside NM.

2 Instantons and probability

An instanton is a classical non-trivial solution to equations of motion in \mathbb{E}^4 with finite, non-zero action S . We recall the main steps of the instanton approach in the field theory [26]. The classical scalar field $\phi(x_j)$ with density $\Pi(\phi)$ of potential energy $V(\phi)$ is given by Lagrangian $L(\phi, x_j) = 1/2 \sum_i (\nabla_i \phi)^2 - \Pi(\phi)$. In the 4D Minkowski spacetime $\mathbb{M}^{3,1}$ the Euler-Lagrange equation under spherical symmetry reads

$$\frac{\partial^2 \phi}{\partial \tau^2} - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi}{\partial r} \right) + \frac{\partial \Pi}{\partial \phi} = 0, \quad (1)$$

where $\tau = ct$. It has to be supplemented with boundary and initial conditions. In the 4D Euclidean space \mathbb{E}^4 the time τ has to be replaced in (1) by $\vartheta = i\tau$,

$$\frac{\partial^2 \phi}{\partial \vartheta^2} + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi}{\partial r} \right) - \frac{\partial \Pi}{\partial \phi} = 0. \quad (2)$$

Then the Euclidean Lagrangian $L_e(\phi, x_j) = 1/2 \sum_i (\nabla_i \phi)^2 + \Pi(\phi)$ gives rise to the Euclidean action $S_e = \int L_e(\phi, x_j) d^3x d\vartheta$. Probability \wp of emergence in $\mathbb{M}^{3,1}$ of the non-trivial solution of equation (1), which is called instanton, is given [26] up to the pre-exponential factor,

$$\wp \propto \exp \left(-\frac{2|S_e|}{\hbar} \right). \quad (3)$$

3 Detonation waves in relativistic hydrodynamics

In the case of isentropic flow, the Lagrangian $L(\phi, x_j)$ of the continuous matter in $\mathbb{M}^{3,1}$ can be taken [24, 9] equal to the pressure $P = W - E$, where W and E denote the enthalpy and energy, respectively. Indeed, such flow allows to introduce [14] a quasipotential Φ such that $\Phi_{,k} = (W/b) u_k$, where b denotes the density of baryon charge and u_k is a four-velocity,

$$u_0 = \frac{1}{\sqrt{1-V^2}}, \quad u_j = \frac{-V_j}{\sqrt{1-V^2}}, \quad j = 1, 2, 3, \quad V^2 = \sum_{1 \leq j \leq 3} V_j^2. \quad (4)$$

The Lagrangian was found in [24], $L(\phi, x_j) = b\sqrt{\Phi_{,k}\Phi^{,k}} - E$. Substitute into the latter the definition of $\Phi_{,k}$ and making use of identity $u_{,k}u^{,k} = 1$ we arrive at equality $L(\phi, x_j) = P$. Such definition is consistent [9] with Euler equation for continuous SM,

$$\frac{1}{W_s} \left(\frac{\partial P_s}{\partial r} + V_s \frac{\partial P_s}{\partial \tau} \right) + \frac{1}{1-V_s^2} \left(\frac{\partial V_s}{\partial \tau} + V_s \frac{\partial V_s}{\partial r} \right) = 0, \quad (5)$$

the energy conservation law,

$$\frac{1}{W_s} \left(\frac{\partial E_s}{\partial \tau} + V_s \frac{\partial E_s}{\partial r} \right) + \frac{1}{1-V_s^2} \left(\frac{\partial V_s}{\partial \tau} + V_s \frac{\partial V_s}{\partial r} \right) + \frac{2V_s}{r} = 0, \quad (6)$$

and baryon charge conservation law,

$$\frac{1}{b_s} \left(\frac{\partial b_s}{\partial \tau} + V_s \frac{\partial b_s}{\partial r} \right) + \frac{1}{1-V_s^2} \left(\frac{\partial V_s}{\partial \tau} + V_s \frac{\partial V_s}{\partial r} \right) + \frac{2V_s}{r} = 0. \quad (7)$$

Velocity $V_s(r, \tau)$ in equations (5, 6, 7) denotes the radial velocity of spherical flow of SM and by b_s the density of baryon charge in SM. These equations have to be supplemented with equation of state for SM [27] and for NM [15],

$$P_s = \frac{E_s - E_o}{3}, \quad \text{and} \quad P_n(\rho_n) = B \rho_n^{5/3}, \quad B = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_n^{8/3}}, \quad (8)$$

Bearing in mind the similarity between equations (5) and (6) find the relationship between two functions $b_s(r, \tau)$ and $W_s(r, \tau)$. According to definition $W_s = E_s + P_s$ and formula (8) we get, $dE_s = 3/4 dW_s$.

Substituting the latter into (6) we obtain

$$\frac{3}{4} \frac{1}{W_s} \left(\frac{\partial W_s}{\partial \tau} + V_s \frac{\partial W_s}{\partial r} \right) + \frac{1}{1-V_s^2} \left(\frac{\partial V_s}{\partial \tau} + V_s \frac{\partial V_s}{\partial r} \right) + \frac{2V_s}{r} = 0. \quad (9)$$

By comparison (6) and (9) we arrive at relationship $b_s(r, \tau) = W_s^{3/4}(r, \tau)$.

As in the classical theory of detonation [14], write self-similar equations (5, 6) assuming that the velocity $V_s(r, \tau)$ is depending on r and τ only through the variable $\xi = r/\tau$ with velocity' dimension, $V_s = V_s(\xi)$. Such equations might be derived by replacing the differential operators in (5, 6),

$$\frac{\partial}{\partial \tau} \rightarrow -\frac{\xi}{\tau} \frac{d}{d\xi}, \quad \frac{\partial}{\partial r} \rightarrow \frac{1}{\tau} \frac{d}{d\xi},$$

i.e.,

$$\left[\frac{1}{C_{so}^2} \left(\frac{V_s - \xi}{1 - \xi V_s} \right)^2 - 1 \right] \times \frac{dV_s}{d\xi} = \frac{2V_s(1 - V_s^2)}{\xi(1 - \xi V_s)}, \quad \frac{1}{E_s} \frac{dE_s}{d\xi} = \frac{4(\xi - V_s)}{(1 - \xi V_s)(1 - V_s^2)} \times \frac{dV_s}{d\xi}. \quad (10)$$

where C_{so} denotes a speed of sound in SM (in units of speed of light c), $C_{so} = \sqrt{\partial P_s / \partial E_s} = 1/\sqrt{3}$.

Both functions $V_s(\xi)$ and $E_s(\xi)$ are odd. Indeed, by replacing $V_s \rightarrow -V_s$, $E_s \rightarrow -E_s$, $\xi \rightarrow -\xi$ in (10) we arrive to the same equations. Boundary conditions (BC) for equations (10) have to be given at DW front where NM–SM transition occurs and the flux density of the energy-momentum tensor and the flux density of the baryon charge are conserved. According to Zeldovich' *normal detonation law* [14] the DW front propagates w.r.t the SM with a speed of sound.

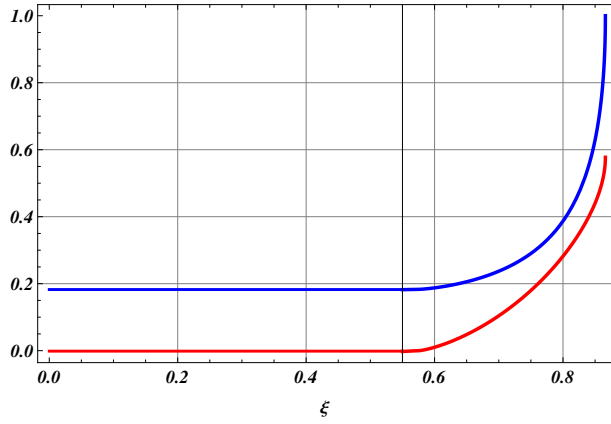


Figure 1: Plots of the functions $V_s(\xi)$ (red curve) and $E_s(\xi)/3E_n$ (blue curve).

When the NM density reaches its value $\rho_* \simeq 10^{15} \text{ g/cm}^3$ and the spontaneous birth of strange quarks proceeds, the NM remains yet non-relativistic. Indeed, in accordance with (8) its pressure reaches $P_n \simeq 6 \cdot 10^{27} \text{ J/cm}^3$ that is much smaller than $E_n = \rho_* c^2 \simeq 10^{29} \text{ J/cm}^3$, i.e., $P_n \ll E_n$. According to [25] the density of baryon charge in NM $b_n = b_s \sqrt{6}$. Assuming that $E_s \gg E_0$, the BCs at the NM–SM front were derived in [25] with ξ_0 denoting the velocity of DW w.r.t. NM,

$$V_s(\xi_0) = 1/\sqrt{3}, \quad E_s(\xi_0) = 3E_n, \quad \xi_0 = \sqrt{3}/2. \quad (11)$$

In Figure 1 we present the plots of the functions $V_s(\xi)$ and $E_s(\xi)/3E_n$ calculated numerically.

4 Instantons in relativistic hydrodynamics

We solve the Euclidean analogue of self-similar equations (10) and use it to calculate the Euclidean action S_e . Introduce a new variable, $\vartheta = i\tau$ which lead to new self-similar variable, $\zeta = r/\vartheta = -i\xi$ and

new velocity function $U_s = dr/d\vartheta = -iV_s$. Substitute it into (10) and get

$$\left[1 + \frac{1}{C_s^2} \left(\frac{U_s - \zeta}{1 + \zeta U_s} \right)^2 \right] \times \frac{dU_s}{d\zeta} = -\frac{2U_s(1 + U_s^2)}{\zeta(1 + \zeta U_s)}, \quad \frac{d \ln P_s}{d\zeta} = \frac{4}{1 + U_s^2} \frac{U_s - \zeta}{1 + \zeta U_s} \times \frac{dU_s}{d\zeta}, \quad (12)$$

where E_s was replaced in (10) by $3P_s$ since $E_s \gg E_o$. Making use of oddness property write BCs for equations (12) as follows, $U_s(\xi_0) = 1/\sqrt{3}$, $P_s(\xi_0) = E_n$. The function $U_s(\zeta)$ has a singular point $\zeta = 0$ and therefore it is convenient to introduce its inverse $\Psi(\zeta) = 1/U_s(\zeta)$ satisfying equation,

$$\frac{d\Psi}{d\zeta} = \frac{2}{\zeta} \frac{(1 + \Psi^2)(\zeta + \Psi)}{3(1 - \zeta\Psi)^2 + (\zeta + \Psi)^2}, \quad \Psi(\xi_0) = \sqrt{3}. \quad (13)$$

Equation (13) has no analytical solution in the range $[0, \sqrt{3}/2]$, however it can be found for $\zeta \ll 1$, where $\zeta, \Psi \rightarrow 0$. It reads, $\Psi(\zeta) = C_1\zeta^{2/3} + \mathcal{O}(\zeta)$, $C_1 > 0$. Substituting $\Psi(\zeta)$ into (12) we obtain

$$\frac{1}{P_s} \frac{dP_s}{d\zeta} = -\frac{8}{\zeta} \frac{1 - \zeta\Psi}{3(1 - \zeta\Psi)^2 + (\zeta + \Psi)^2}, \quad P_s(\xi_0) = E_n. \quad (14)$$

Numerical solutions of equations (13, 14) are presented at Figure 2. The function $P_s(\zeta)$ determines the distribution of pressure in \mathbb{E}^4 and allows to calculate the Euclidean action S_e which enters into (3).

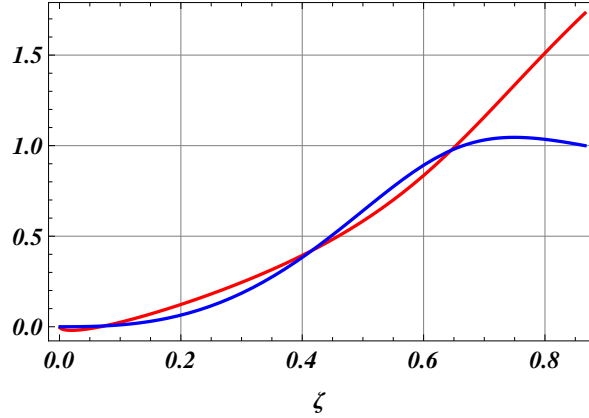


Figure 2: Plots of the functions $\Psi(\zeta)$ (red curve) and $E_n/P_s(\xi)$ (blue curve).

To show this we prove a coincidence of the Lagrangian $L(\phi, x_j)$ with the pressure P_s . Indeed, extend the Lagrangian $L(\phi, x_j)$ analytically over complex time $\tau = \vartheta/i$. Substituting $V_s = iU_s$ into (4) we find that $u_{,k}u^{,k} = 1$ in \mathbb{E}^4 as well as in $\mathbb{M}^{3,1}$ space. Thus, we obtain $L_e(\phi, x_j) = P_s$.

Start with Lorentzian action S for self-similar DW. Make use of equality $L(\phi, x_j) = P$ and find

$$S = \frac{4\pi}{c} \int_0^{cT_1} \int_0^{R(T_1)} P_s \left(\frac{r}{\tau} \right) r^2 dr d\tau = \frac{4\pi}{c} \int_0^{\xi_0} P_s(\xi) \xi^2 d\xi \int_0^{cT_1} \tau^3 d\tau. \quad (15)$$

The radius $R(T_1) = \xi_0 c T_1$ of SM sphere determines the DW front which propagates toward NM with velocity $\xi_0 c$ during a time T_1 in such a way that a pressure vanishes at the front $P_s(r) = 0$, $r > R$. The r.h.s. in (15) is written by rescaling to the self-similar variable ξ .

Write the Eucledian action S_e by replacing $\tau \rightarrow \vartheta/i$ and $P_s(\xi) \rightarrow P_s(\zeta)$ and normalizing $P_s(\zeta) = P_s(\zeta_0)p(\zeta)$ where $P_s(\zeta_0) = P_s(\xi_0)$ and $p(\zeta_0) = 1$. Substitute the last into (15) and integrate numerically,

$$S_e = -i\pi c^3 P_s(\xi_0) T_1^4 J(\xi_0), \quad J(\xi_0) = \int_0^{\xi_0} p(\zeta) \zeta^2 d\zeta \simeq 0.6514. \quad (16)$$

Estimate S_e by following consideration. In NS with the total mass $M \geq 1.5\odot$ and density $\rho_* \simeq 10^{15} \text{ g/cm}^3$ the spontaneous conversion of NM to SM is expected when a density of baryon charge reaches $n_A \simeq 6 \cdot 10^{38} \text{ cm}^{-3}$. Since the spherical DW front propagates with velocity $\xi_0 c$ then the total baryon charge grows as $A = 4\pi/3 n_A (\xi_0 c T_1)^3$, i.e.,

$$T_1 = \frac{1}{\xi_0 c} \sqrt[3]{3A/(4\pi n_A)}. \quad (17)$$

Keep in mind an equality $P_s(\xi_0) \simeq 3m_n n_A c^2$ which follows from (11), and combine it with (16),

$$\frac{|S_e|}{\hbar} = \frac{4r_B A^{4/3}}{3\lambda_c} J(\xi_0). \quad (18)$$

where m_n denotes a mass of neutron, $\lambda_c = \hbar/m_n c \simeq 2.1 \cdot 10^{-14} \text{ cm}$ stands for the Compton wavelength and $r_B \simeq \sqrt[3]{3/(4\pi n_A)}$ denotes an average distance between neutrons in NS core (for $n_A \simeq 6 \cdot 10^{38} \text{ cm}^{-3}$ we have $r_A \simeq 7.4 \cdot 10^{-14} \text{ cm}$).

In the WKB approximation a frequency \mathcal{V} of emergence of the DW during the spontaneous conversion of A neutrons to SM reads, $\mathcal{V} = \wp/T_1$ where the barrrier transparency \wp is given in (3). Appearance of the strangelets in NS core gives rise to propagate of DW and leads to explosion of NS. This scenario is realized during the time existence T_2 of NS and allows to estimate the necessary value of A . According to astrophysical observations [28] the largest time existence of NS is approximately 10^6 years but not exceeding the universe age $13.8 \cdot 10^9$ years, i.e., we have $3.15 \cdot 10^{13} \text{ s} < T_2 < 4.35 \cdot 10^{17} \text{ s}$. The entire number of *potential* strangelets in the NS core is given by N_A/A where $N_A = M_c/m_n$ denotes a number of neutrons in the NS core and the mass M_c of core is estimated as 1% of the total NS mass, $M_c \simeq 10^{-2} \cdot 1.5\odot$ [28]. Then a probability \mathbb{P} to have at least one strangelet in core during the time T_2 is dependent on A and reads

$$\mathbb{P}(A) = \frac{N_A}{A} \frac{T_2}{T_1} \exp\left(-\frac{2|S_e|}{\hbar}\right), \quad S_e = S_e(A), \quad T_1 = T_1(A), \quad T_2 = 10^{t(A)}. \quad (19)$$

To find a lower and upper bounds for critical value A_* providing an appearance at least one strangelet in the core let us require $\mathbb{P}(A_*) = 1$. Solving this transcendental equation for the lower and upper bounds of T_2 we get: $23.8 < A_* < 24.61$. These values are pretty close to $A = 20$ used in [12, 18] for calculation of the ground state of strangelets in the framework of the MIT bag model and $A = 16$ taken from space-based particle physics experiments on the Alpha Magnetic Spectrometer [4] during the Space Shuttle Discovery mission in 1998.

5 Concluding Remarks

In the framework of instanton approach we have shown that NS with the core density $\rho_* \simeq 10^{15} \text{ g/cm}^3$ allows to have at least one stable strangelet during the time star existence T_2 , $T_N < T_2 < T_U$, if the baryon number is $A_* = 24$, where $T_N \simeq 10^6$ years and $T_U \simeq 13.8 \cdot 10^9$ years stand for the largest time NS existence and the universe age, respectively. A low value of A_* makes it interesting to compare it with those discussed in literature.

For $2 < A < 6$ quantum chromodynamics strongly suggests complete instability of any strangelets [10]. In [6] the SM is studied for low $A < 10^2$ and large $10^2 < A < 10^7$ baryon numbers. This wide range covers many other values for A discussed in literature: $A \simeq 16 - 40$ [3], $A < 10^2$ [7], $A > 10^2$ [21], $A \simeq 10^3$ [27], $A \simeq 10^2 - 10^4$ [19] and most of these values are substantially larger than A_* . We put forward an agent which may be responsible for the higher A_* in the framework of NM-SM instanton transition.

The mass of equilibrium configuration of cold matter at each central star density ρ_* (g/cm^3) is a damped periodic function of $\ln \rho_*$ [11, 29]. There are two ranges for which these configurations are stable : the white dwarfs with low electron density, $10^5 < \rho_* < 10^8$, and the neutron stars with high density, $10^{14} < \rho_* < \rho^{OV}$ where $\rho^{OV} \simeq 6 \cdot 10^{15}$ denotes the Oppenheimer-Volkoff limit. There are also a number of extrema for ρ_* exceeding ρ^{OV} : such superdense configurations were found in [20], $10^{18} < \rho_* < 10^{20}$, and in [11], $\rho_* > 3 \cdot 10^{21}$. In Figure 3 we show how the admissible values of A do increase once the density of the NS core grows, e.g., for $\rho_* \simeq 10^{19}$ we have $240 < A_* < 250$.

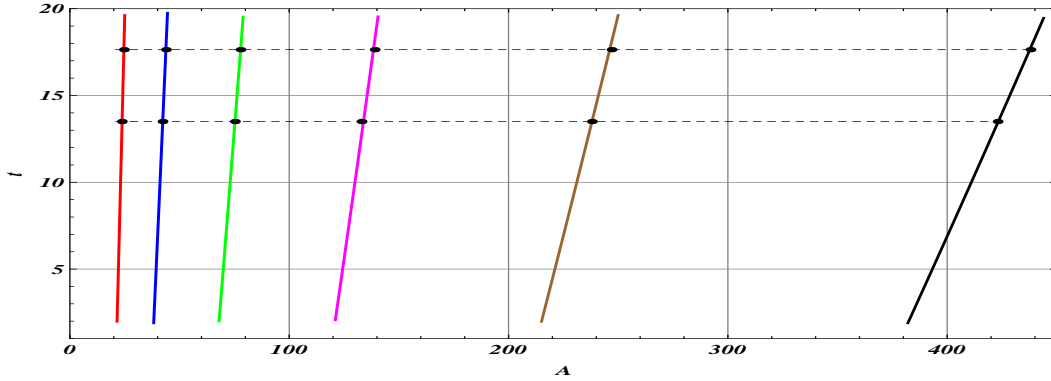


Figure 3: Plots of the functions $t(A)$ defined in (19) for different densities ρ_* (g/cm^3) in the NS core: 10^{15} (red), 10^{16} (blue), 10^{17} (green), 10^{18} (magenta), 10^{19} (brown), 10^{20} (black). Two dashed lines mark two time scales $T_N = 10^{t_n}$, $t_n \simeq 13.5$, and $T_U = 10^{t_u}$, $t_u \simeq 17.64$, and intersect the lines at black points.

In fact, all superdense configurations with $\rho_* > \rho^{OV}$ are metastable due to the acoustic vibrations

[5, 11] propagating in stars with characteristic time $T_a = (\gamma\bar{\rho})^{-1/2}$ where γ denotes a gravitational constant and $\bar{\rho} = M_{NS}/V_{NS}$ denotes an average density of NS of the total mass M_{NS} and volume V_{NS} . E.g., if $\rho_* \simeq 10^{19}$ then according to [11] $\bar{\rho} \simeq 0.25 \cdot 10^{15}$ and finally we have $T_a \simeq 2 \cdot 10^{-4}$ s. Simple calculation by formula (19) with $T_2 = T_a$ and $\mathbb{P}(A_*) = 1$, gives a value of A_* that provides to have at least one strangelet in core during the time T_a , i.e., $A_* \simeq 200$. Suggestions of superdense stars with core density above ρ^{OV} continue to appear in the literature [8, 22].

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